#### ERCIM SNA WG Algorithmic Aspects of Social Network Analysis Guillaume-Jean Herbiet March, 11th 2010, UCD, Dublin

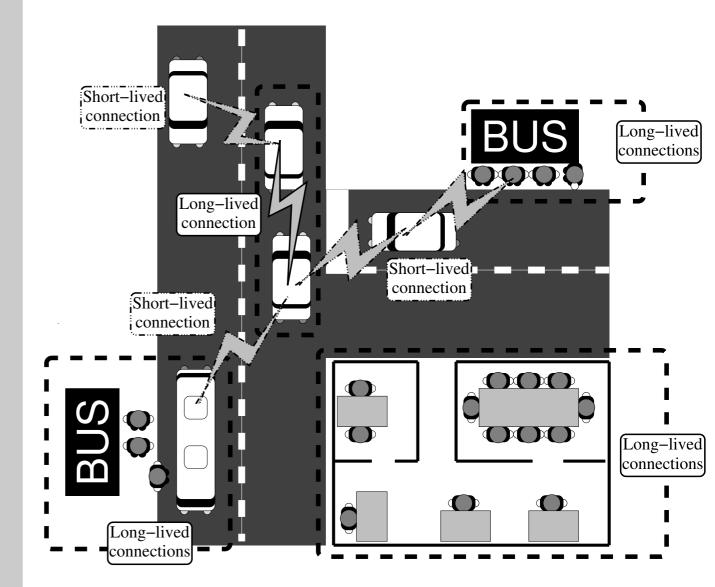


Distributed community detection over dynamic networks using neighborhood similarity



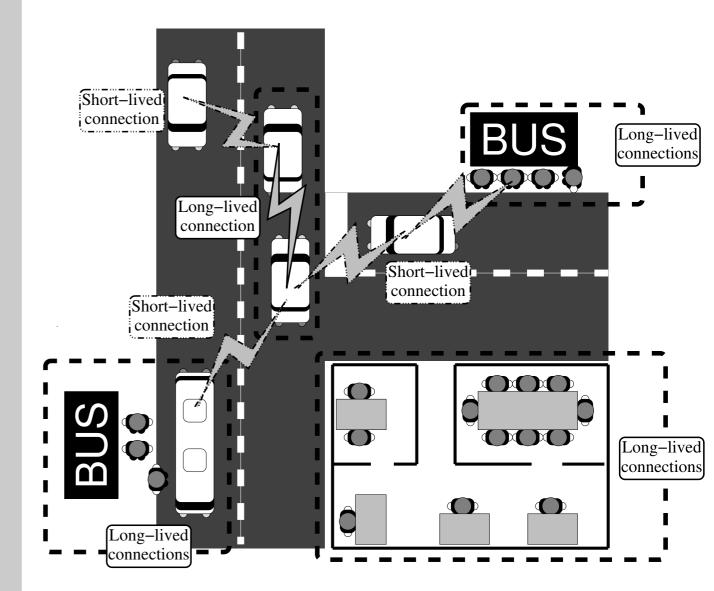


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  - Propagation/environment
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  - Subgraphs of interactive and high-end communications
  - Use weak ties for delay-tolerant global information



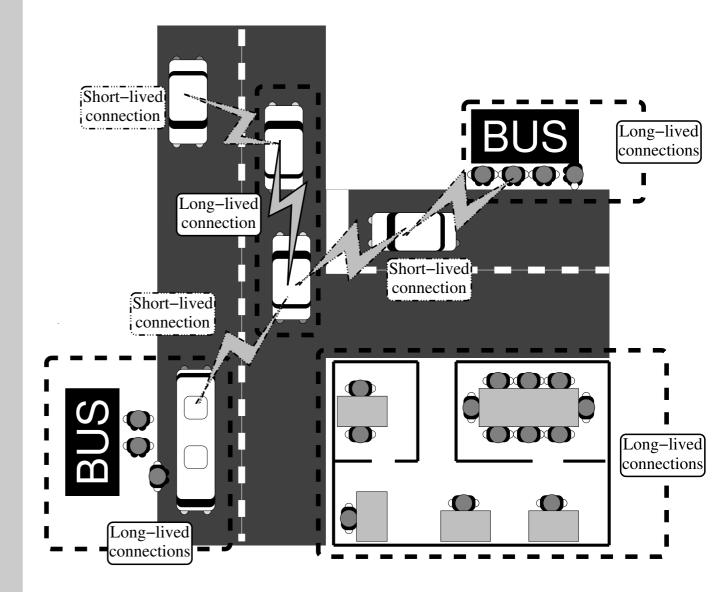


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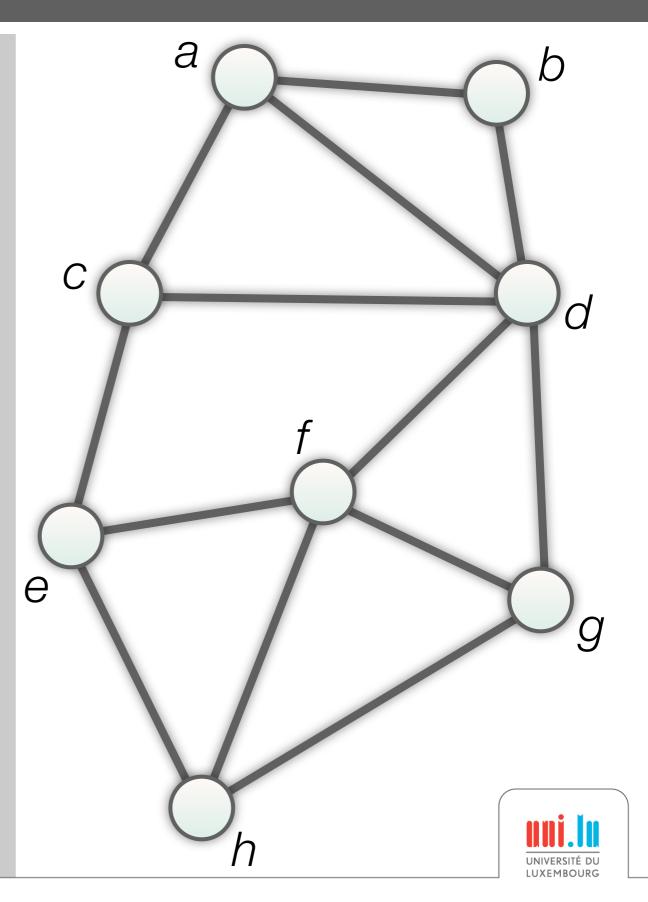
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#### Challenges:

- Dynamic network
- Distributed approach



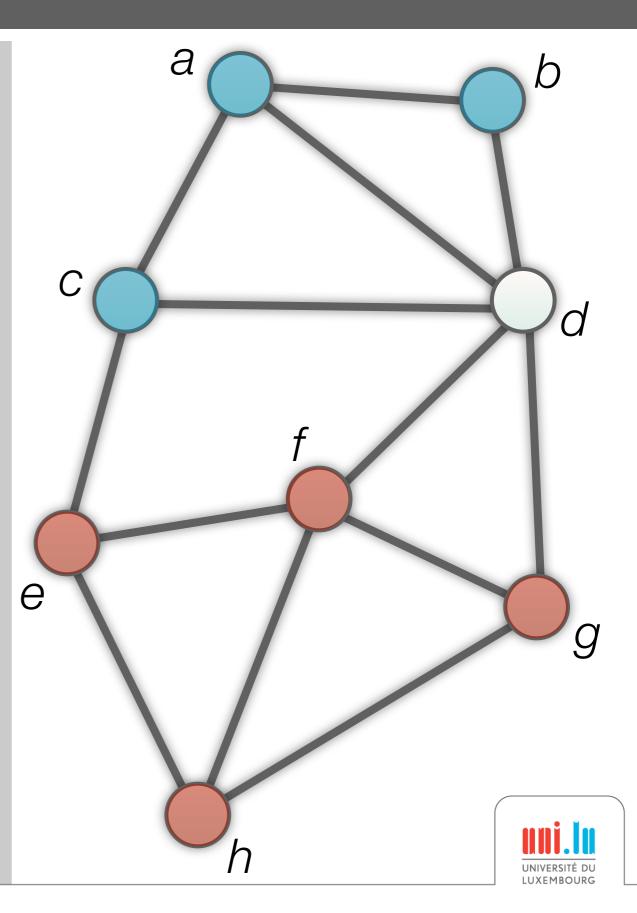




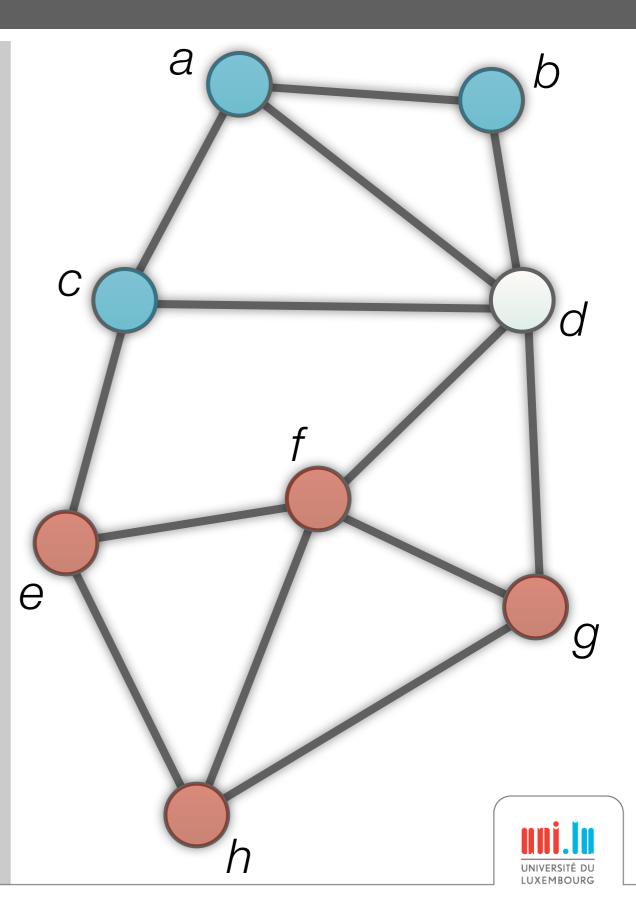
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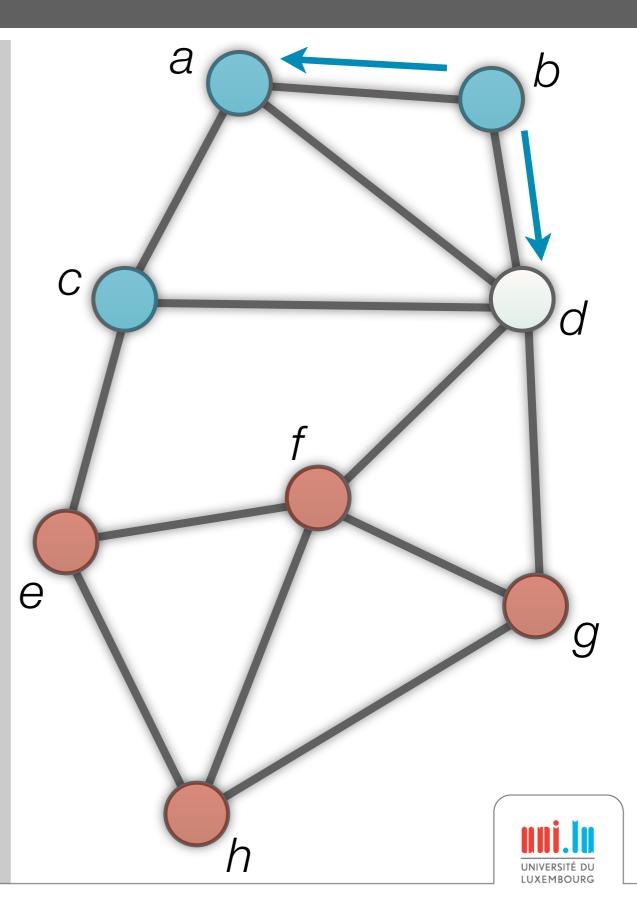
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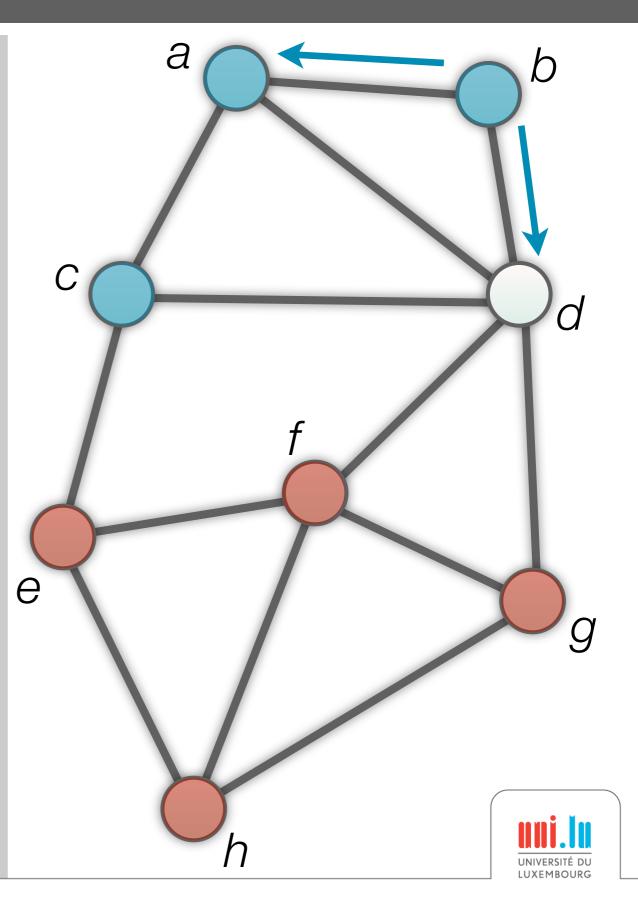
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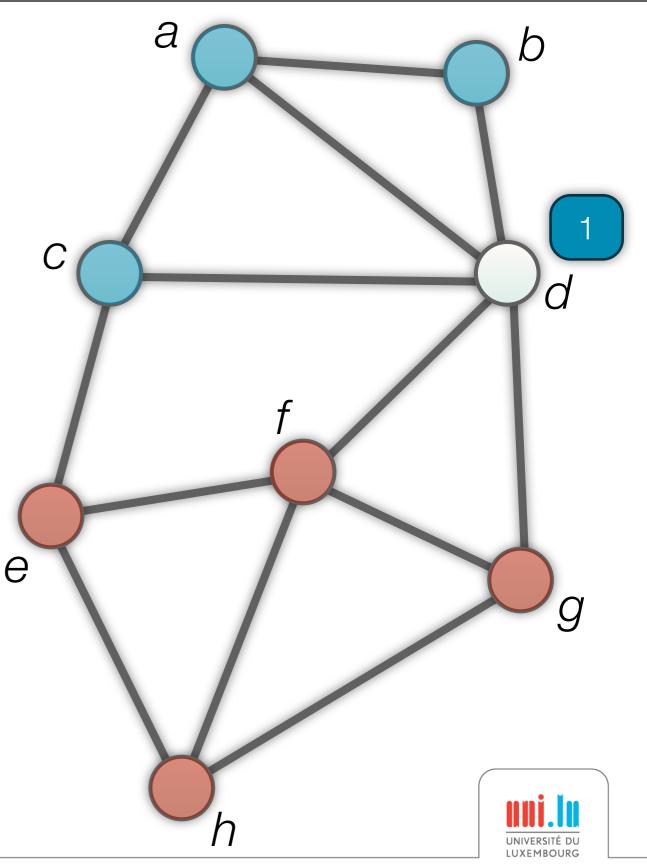
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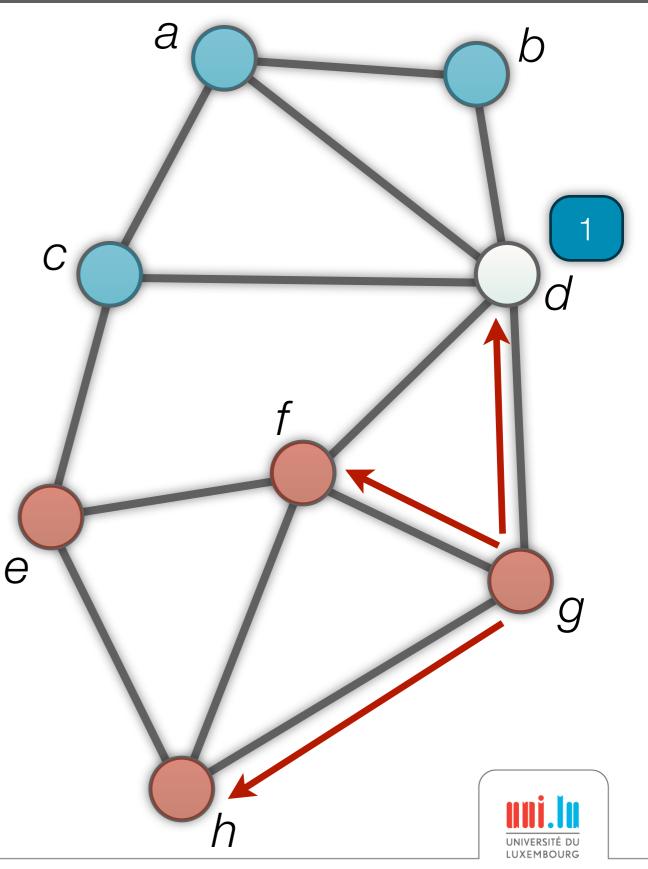
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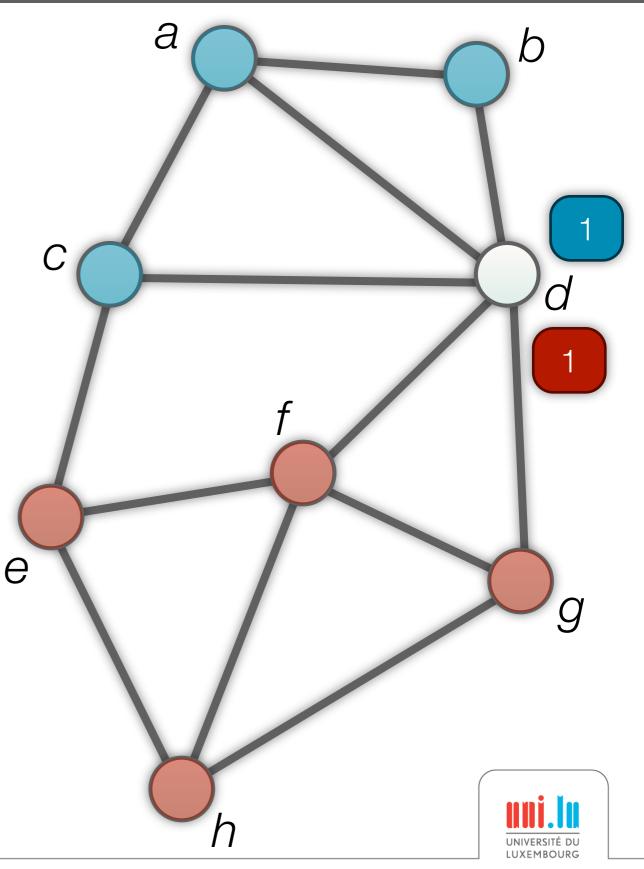
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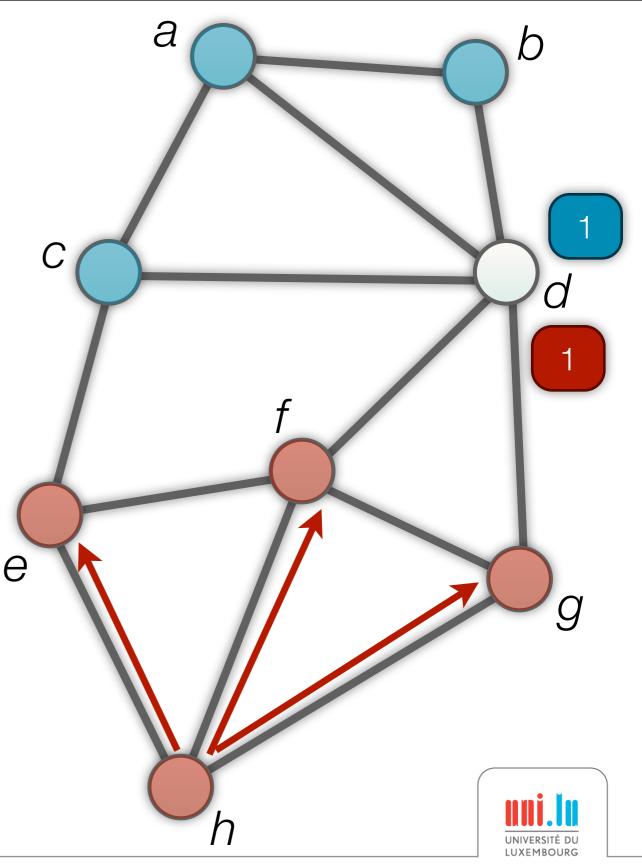
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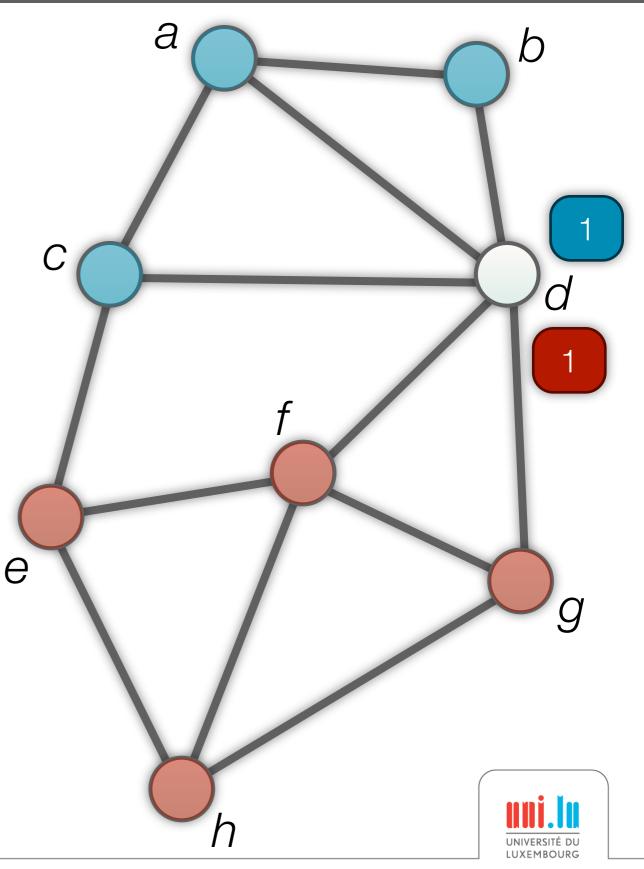
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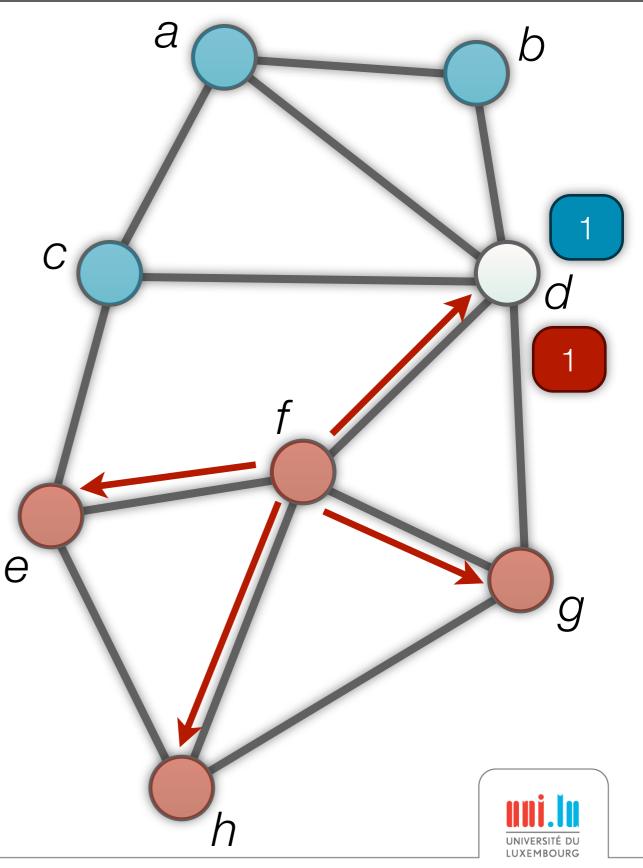
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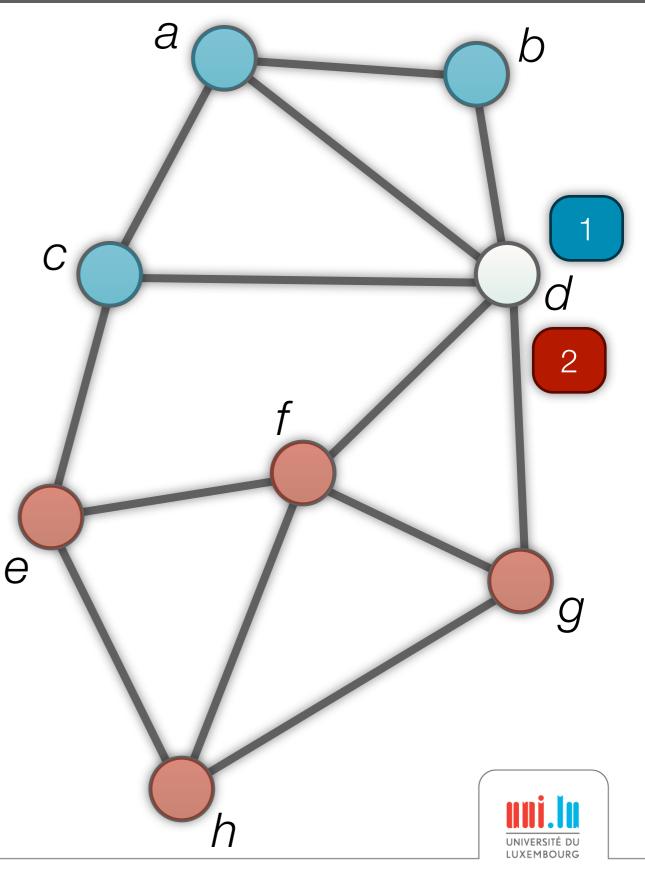
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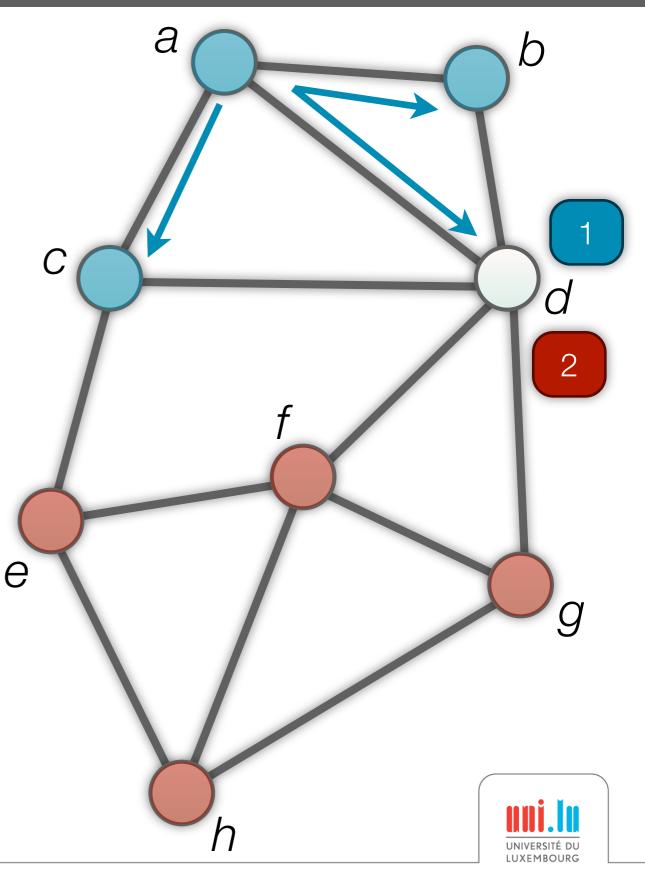
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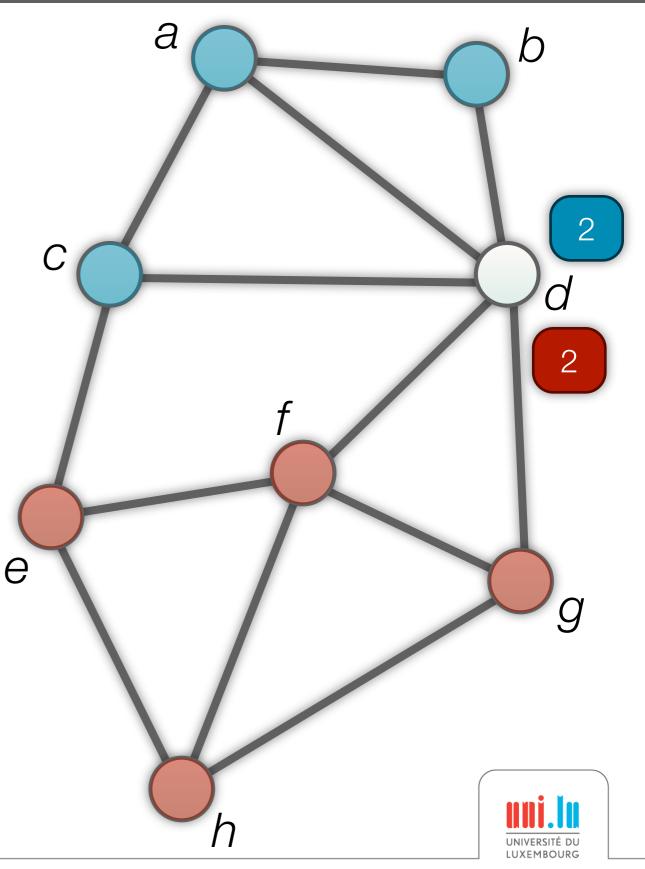
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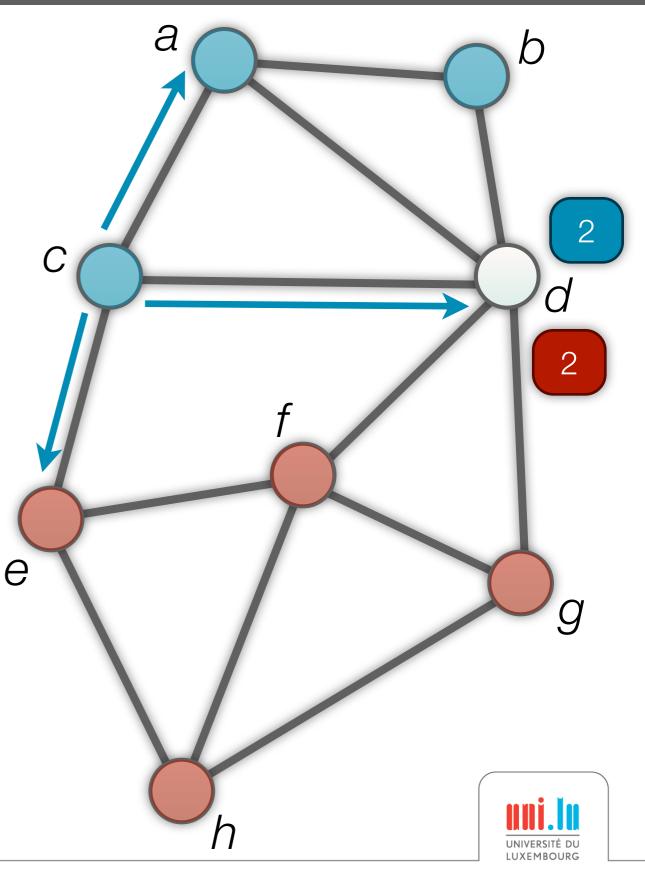
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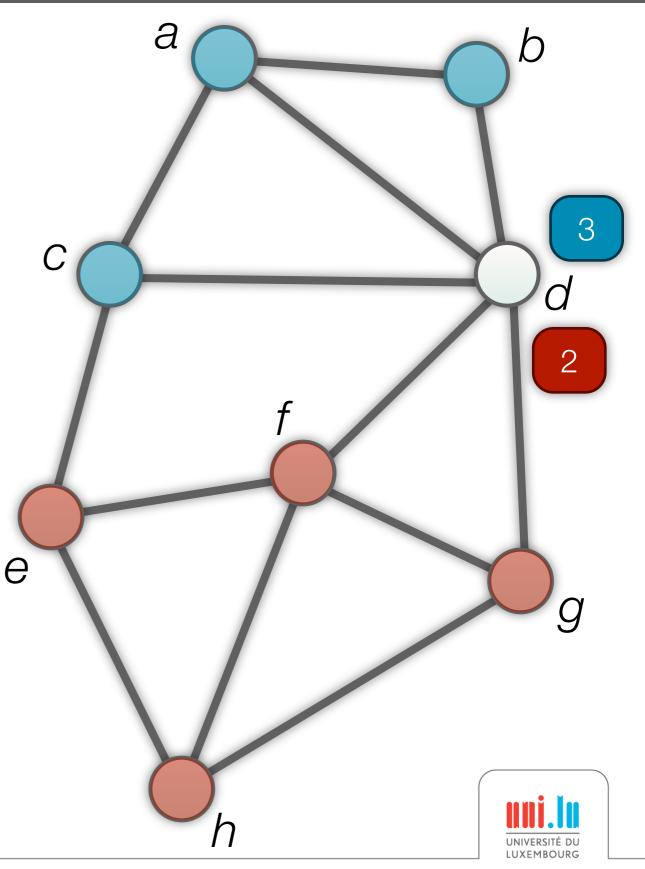
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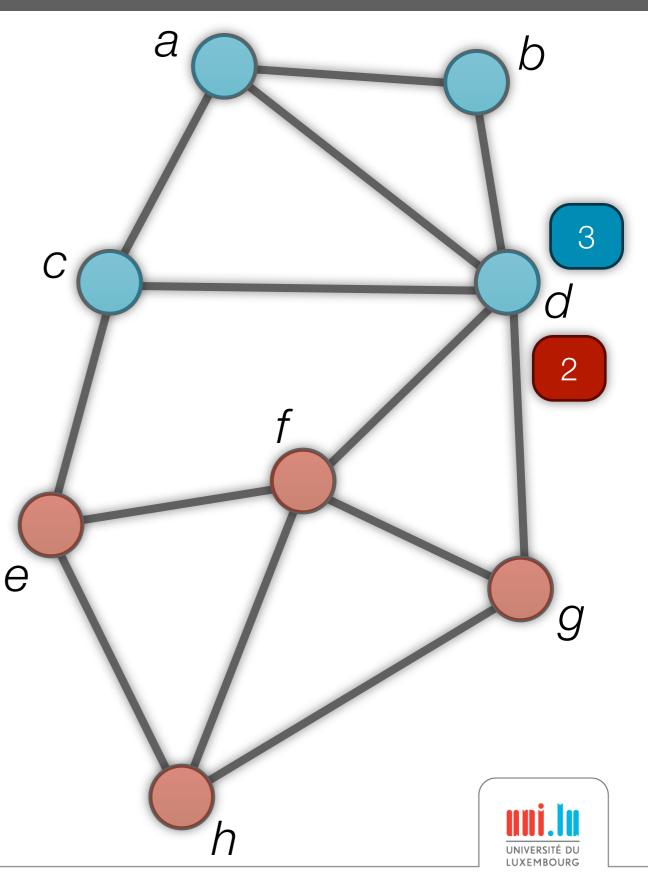
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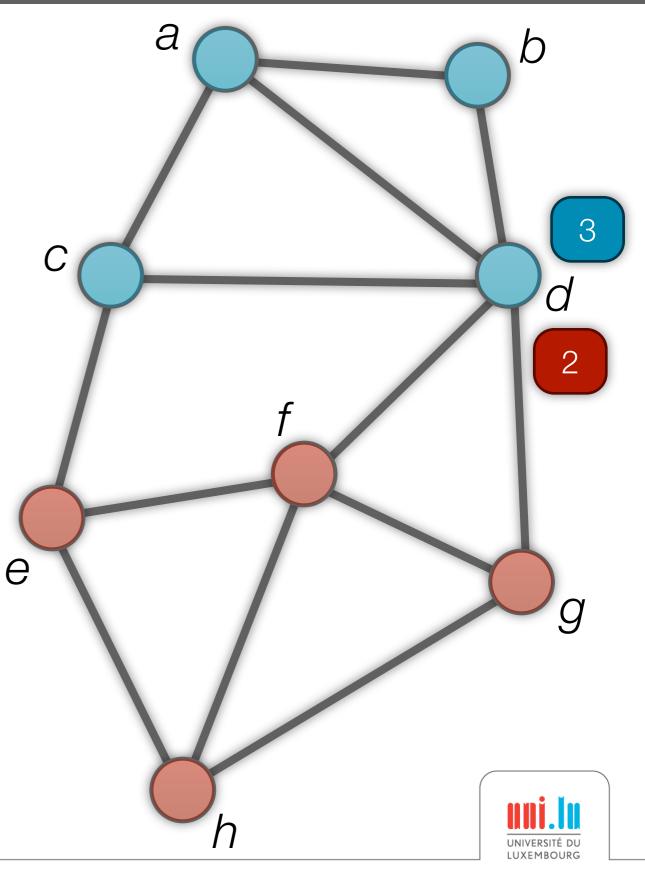
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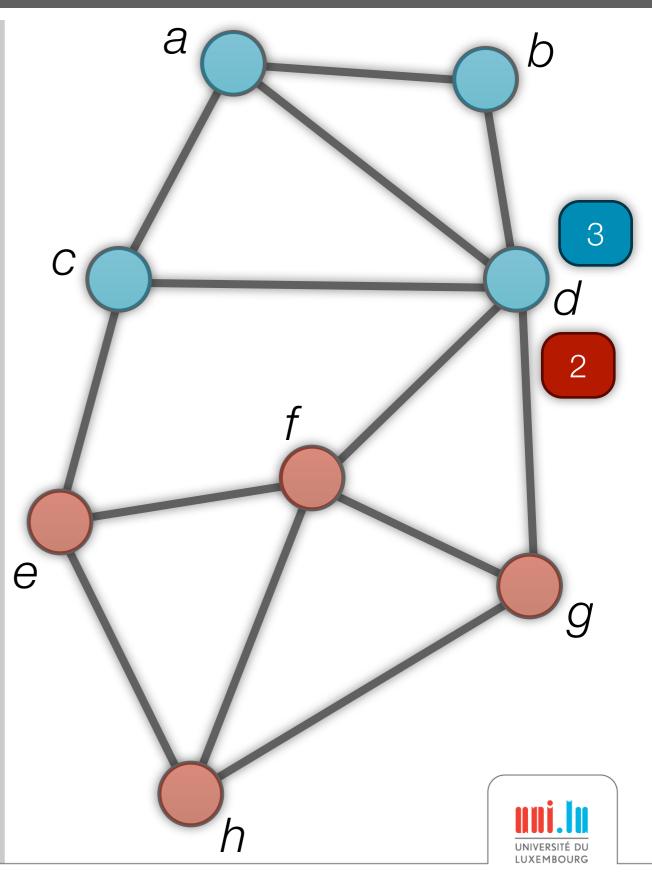
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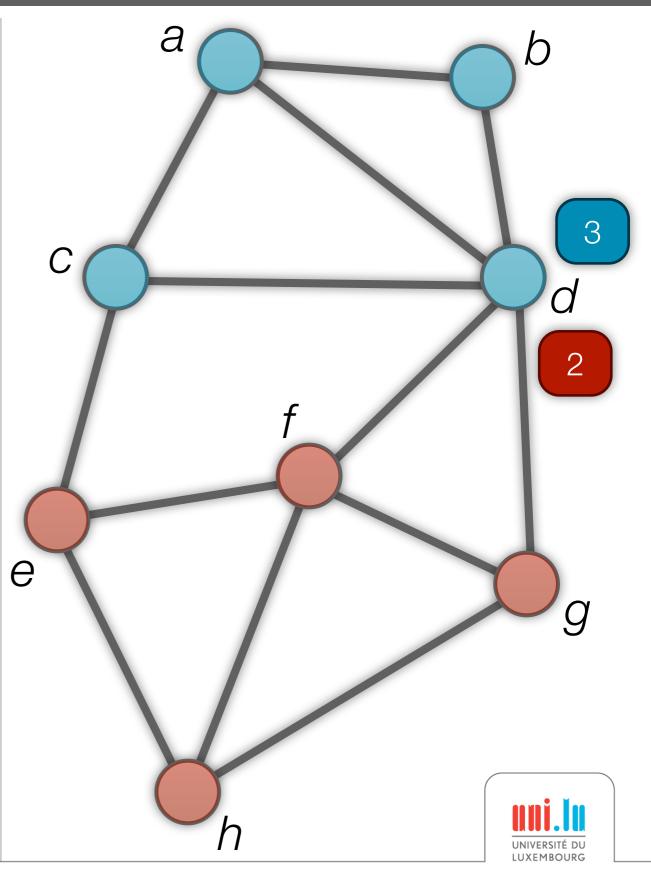
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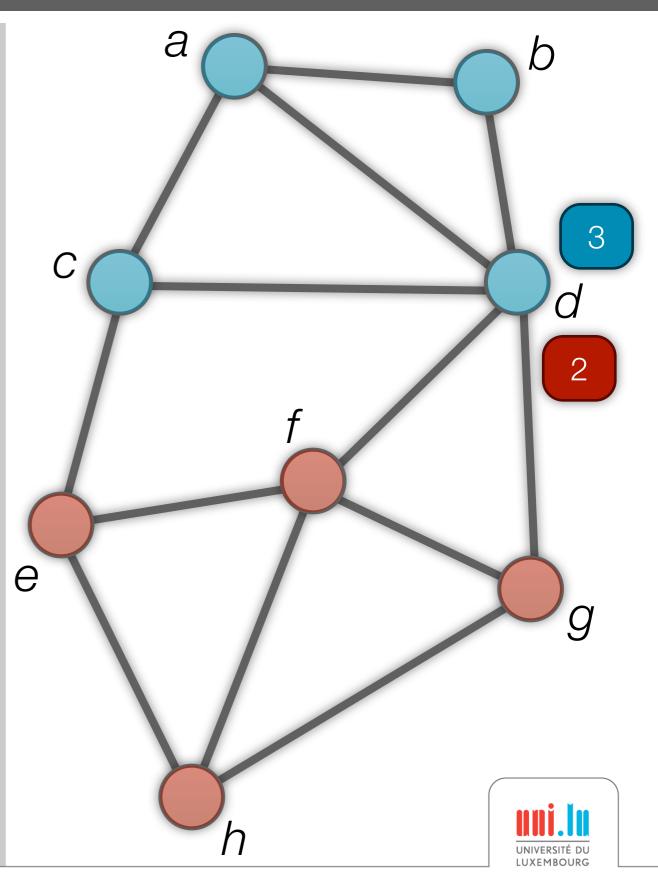
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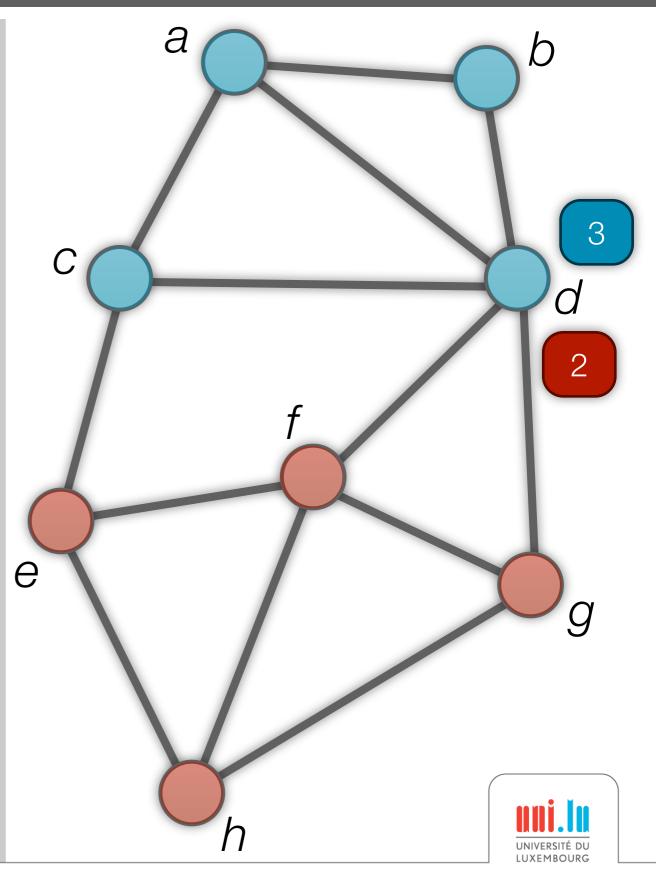
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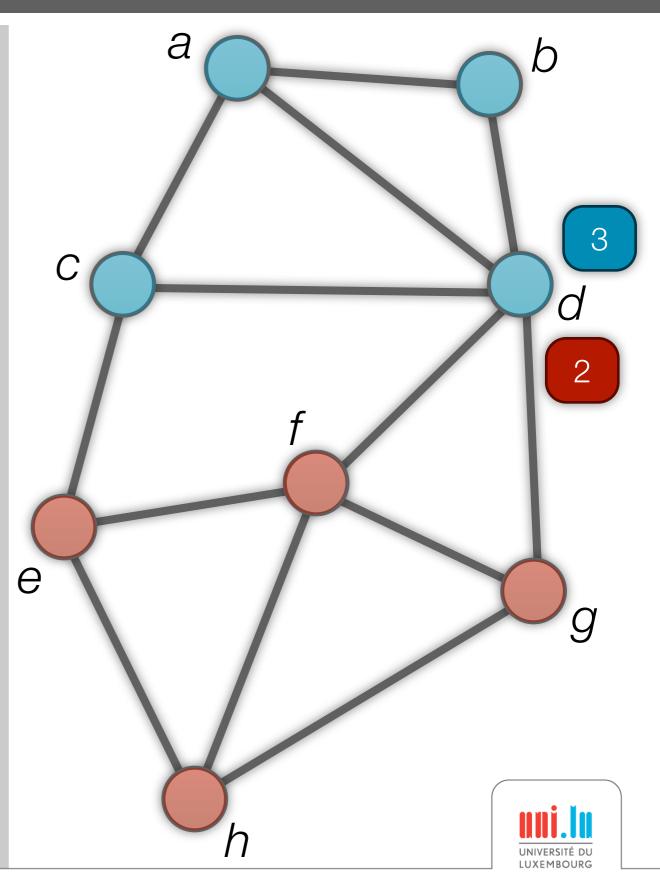
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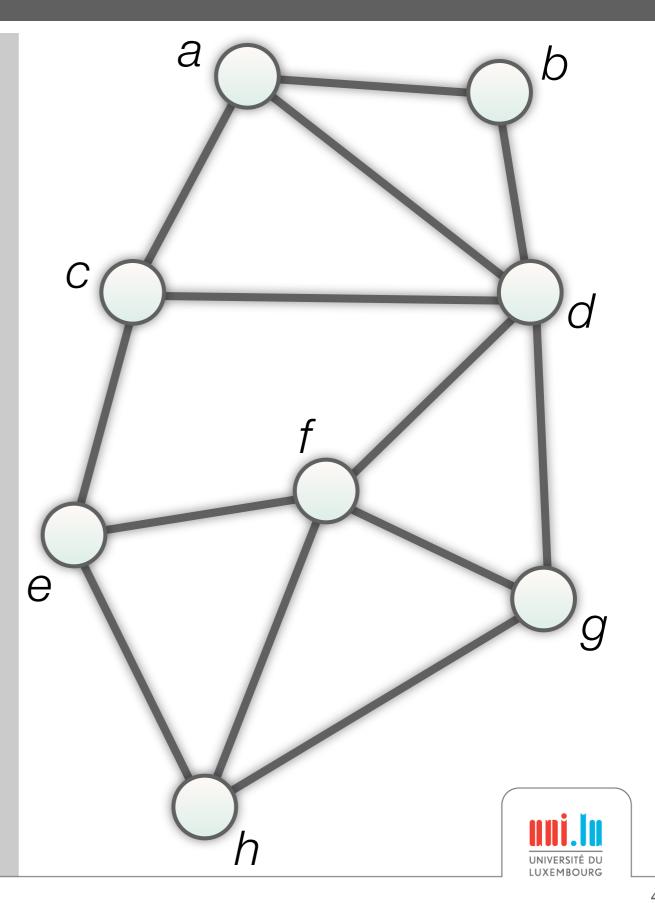
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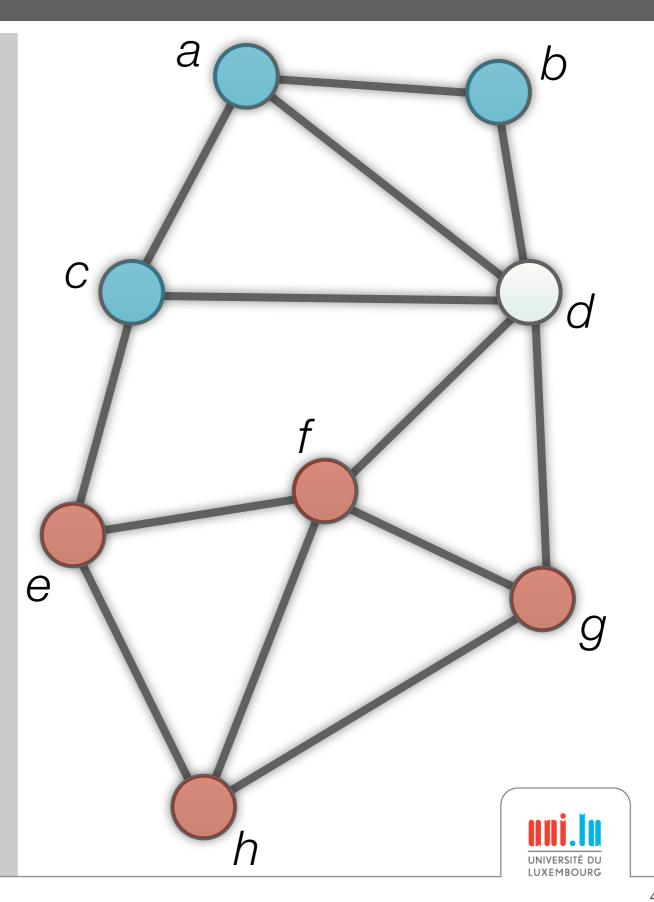
#### Not really focusing on dynamic networks

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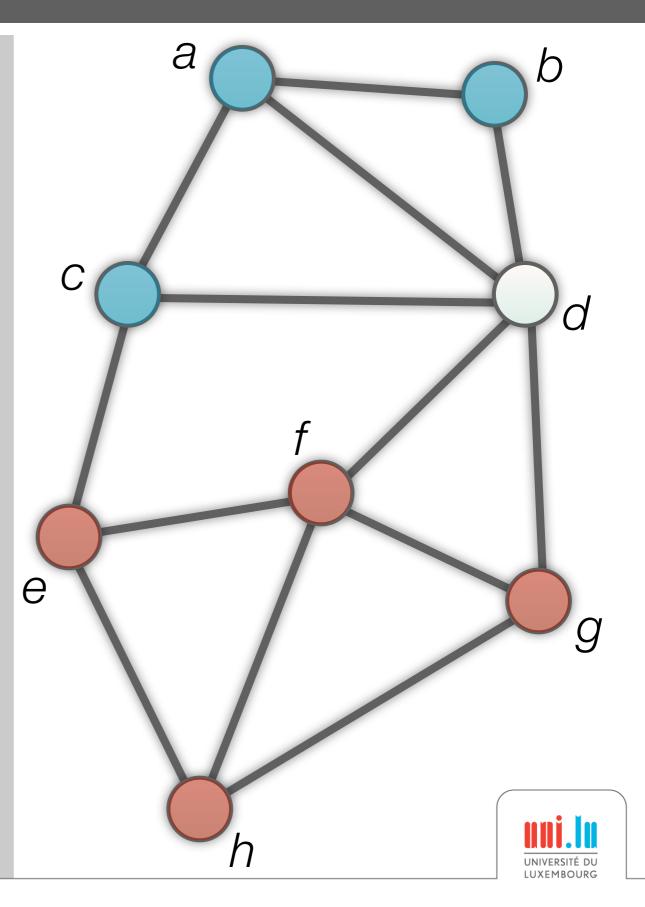
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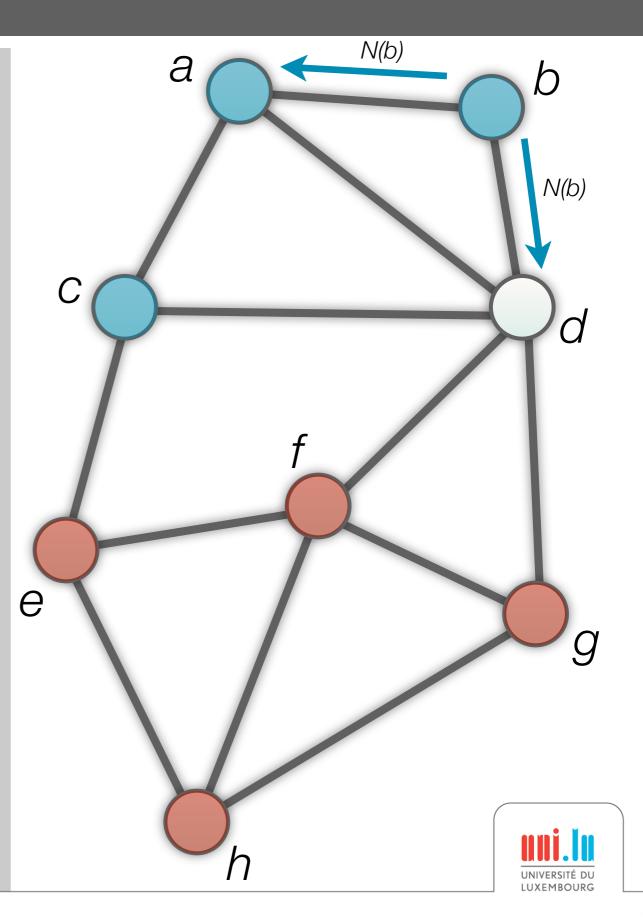


SHARC uses two-hop information



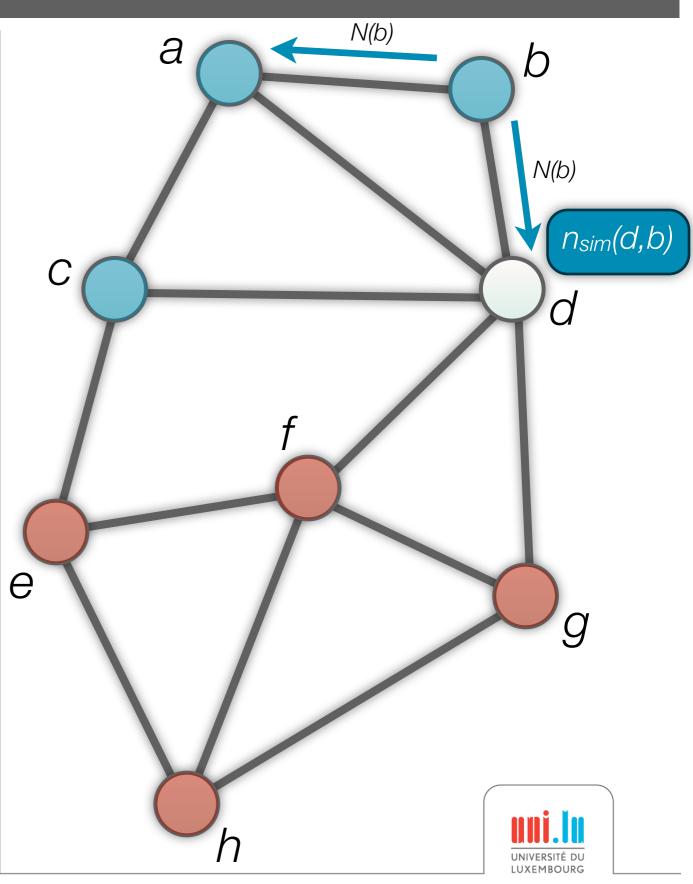
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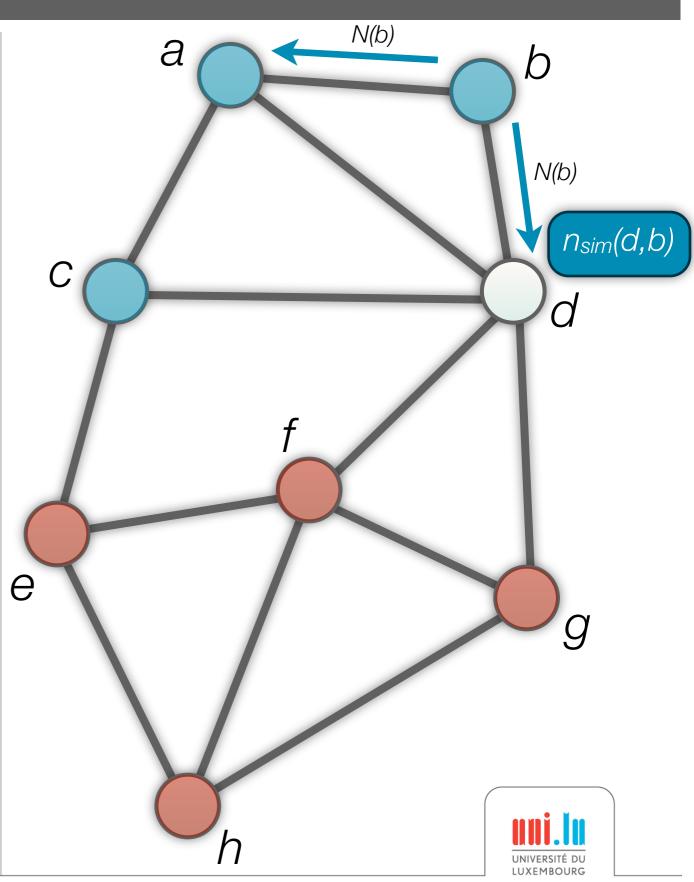
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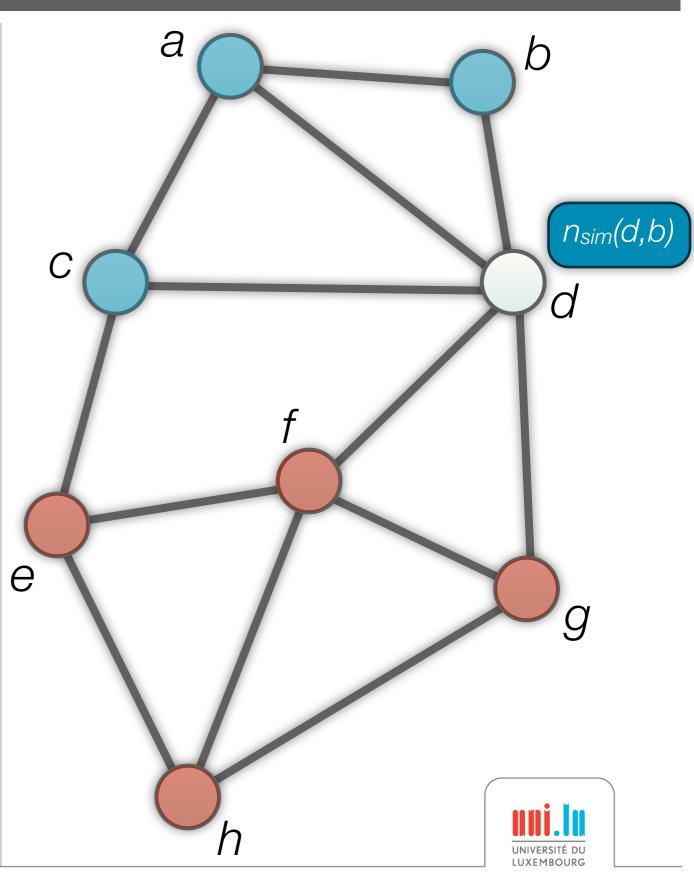
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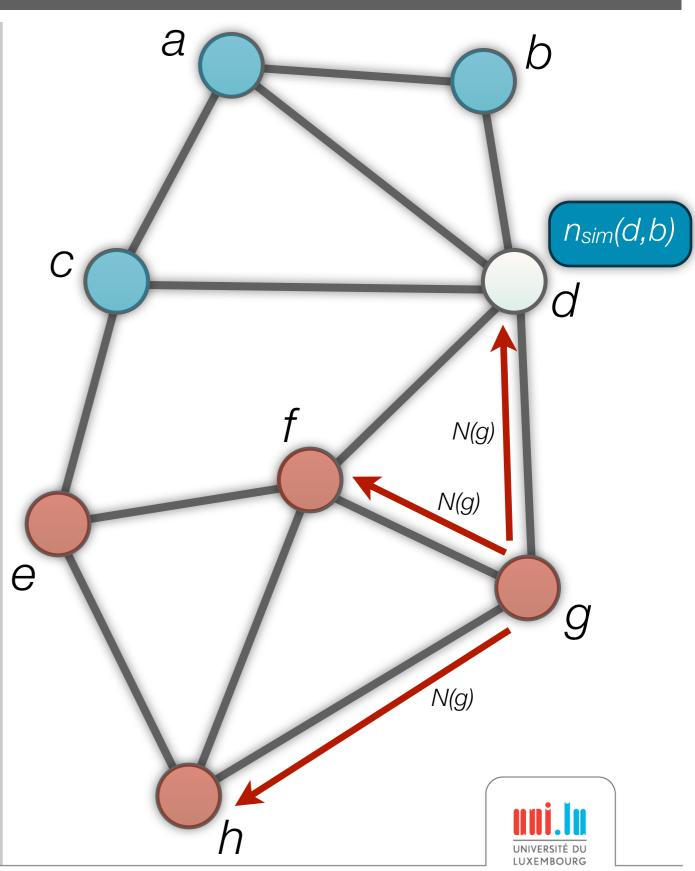
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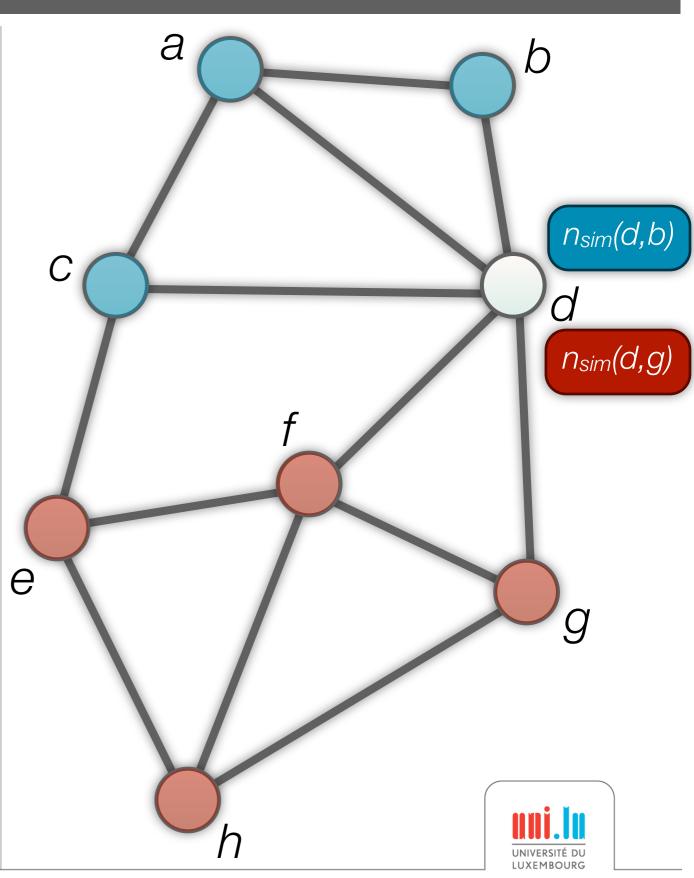
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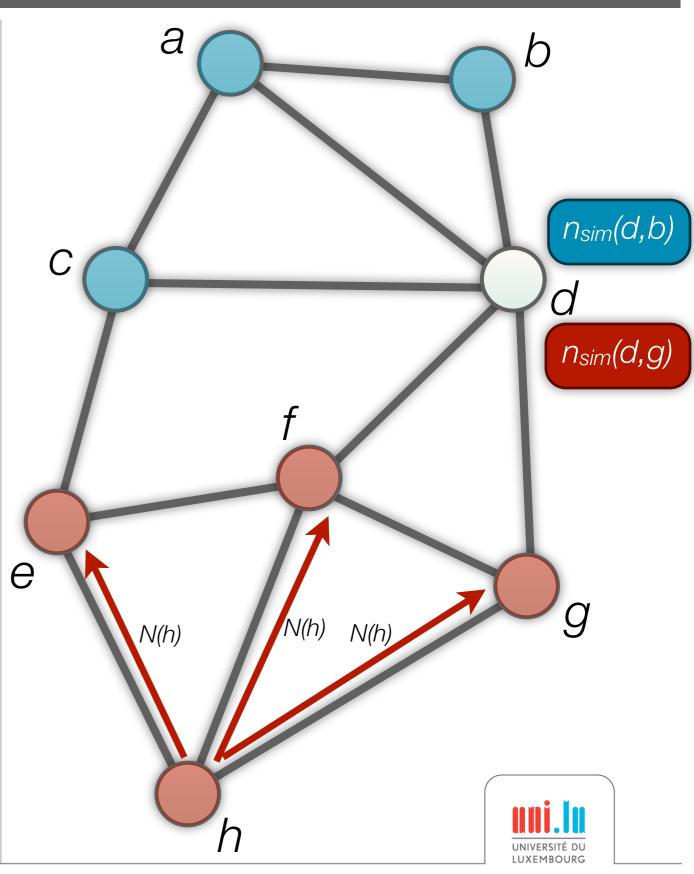
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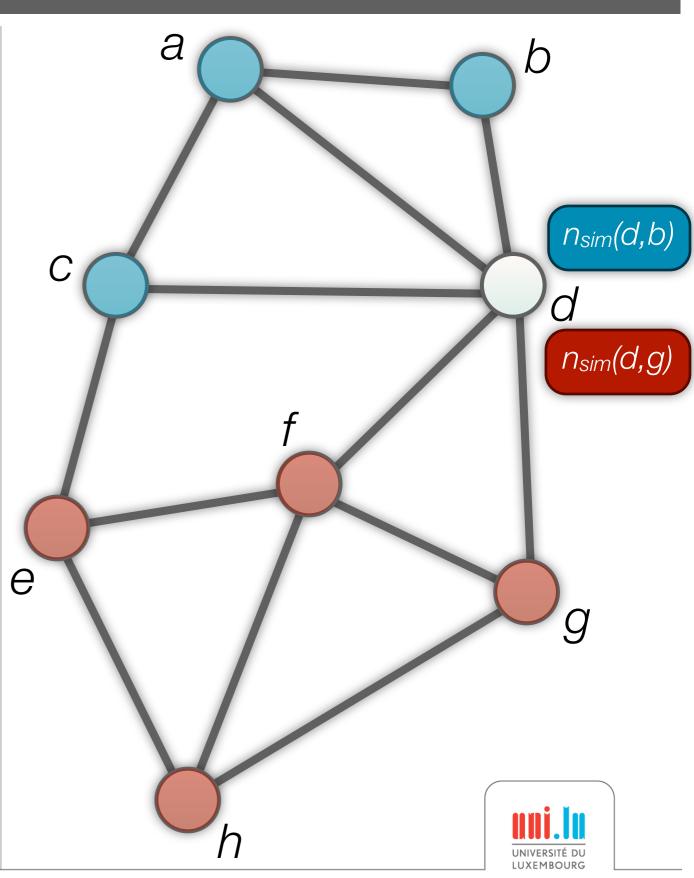
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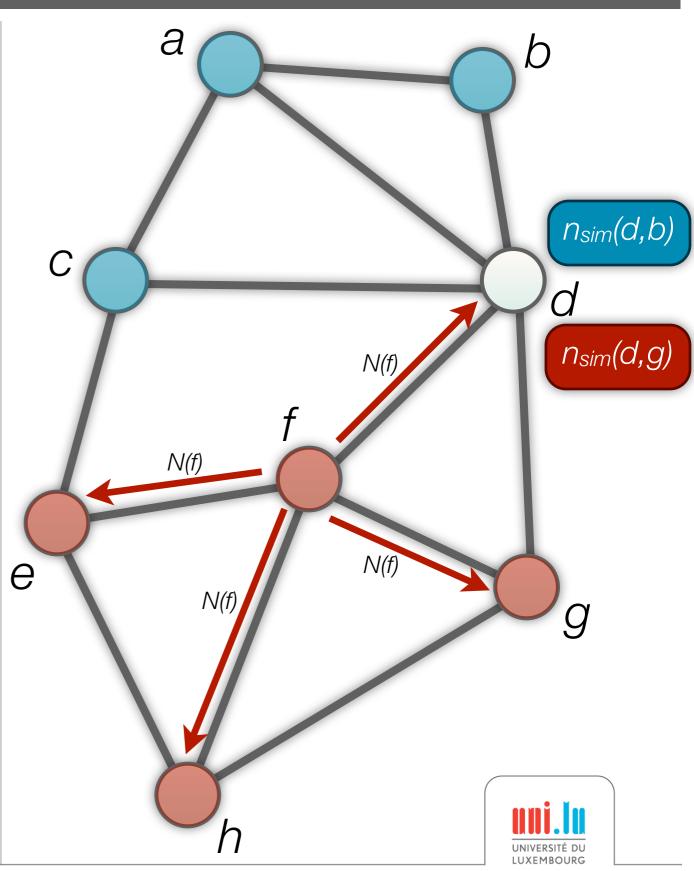
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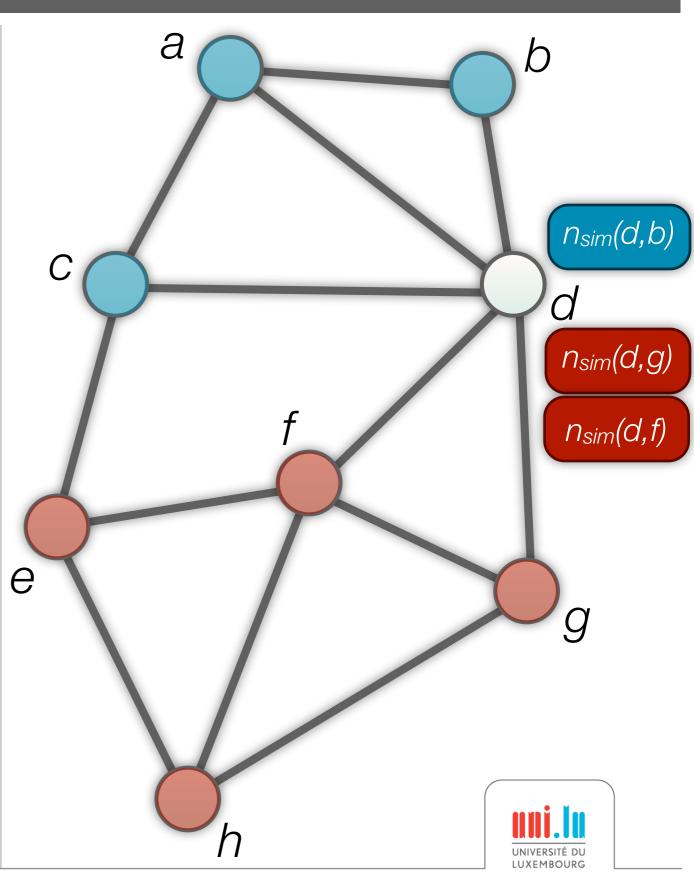
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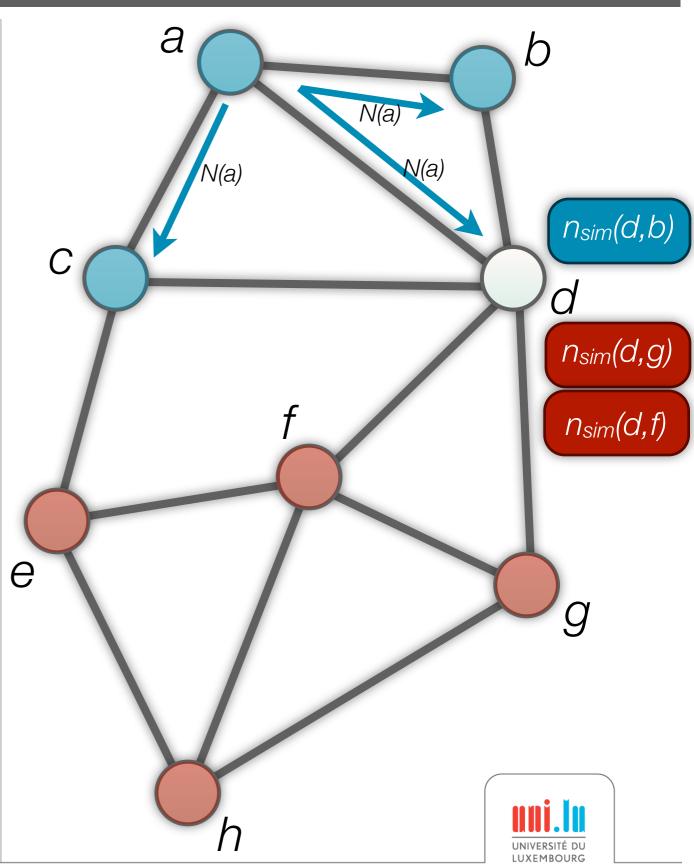
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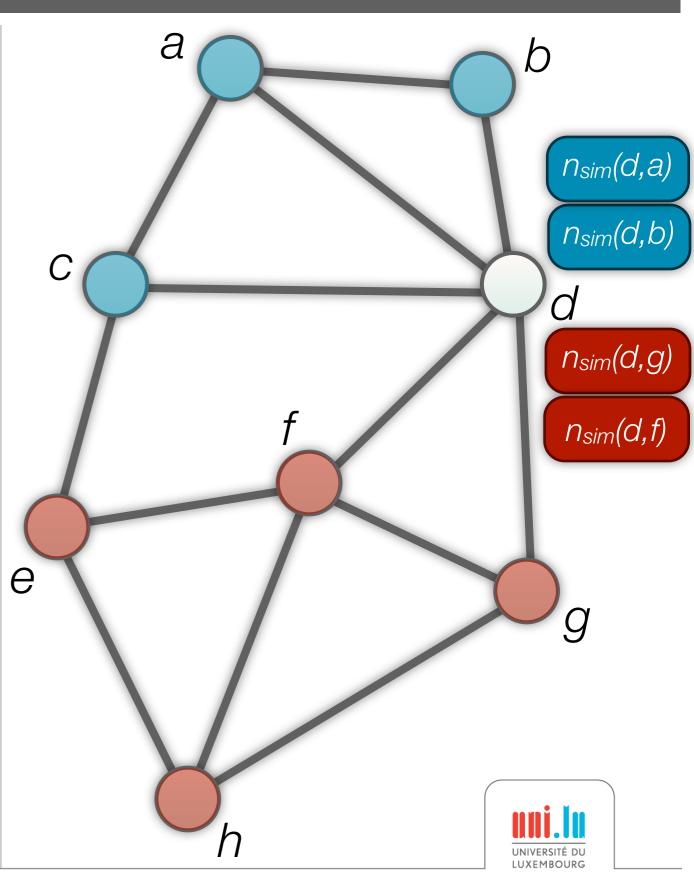
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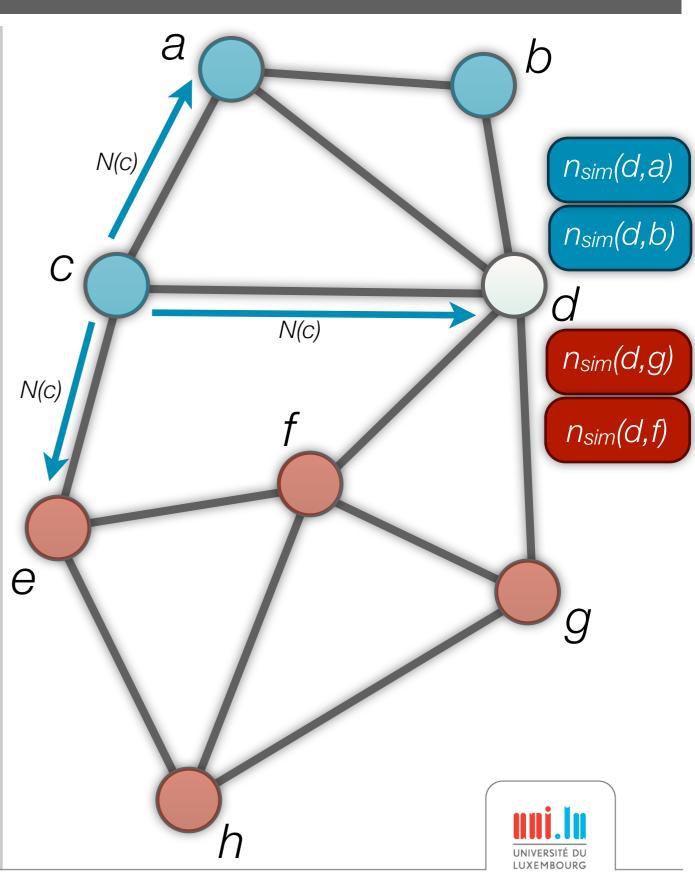
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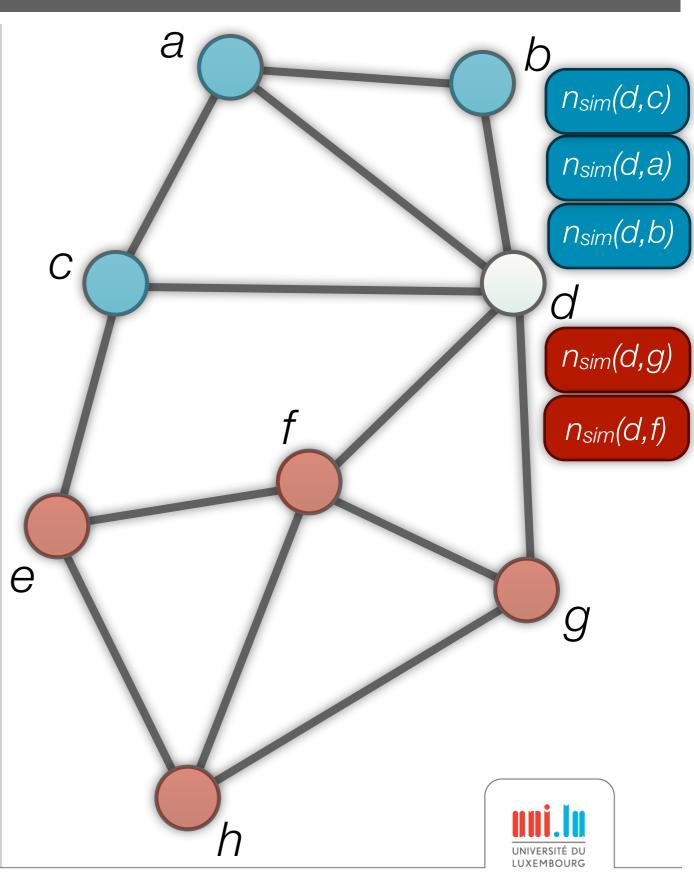
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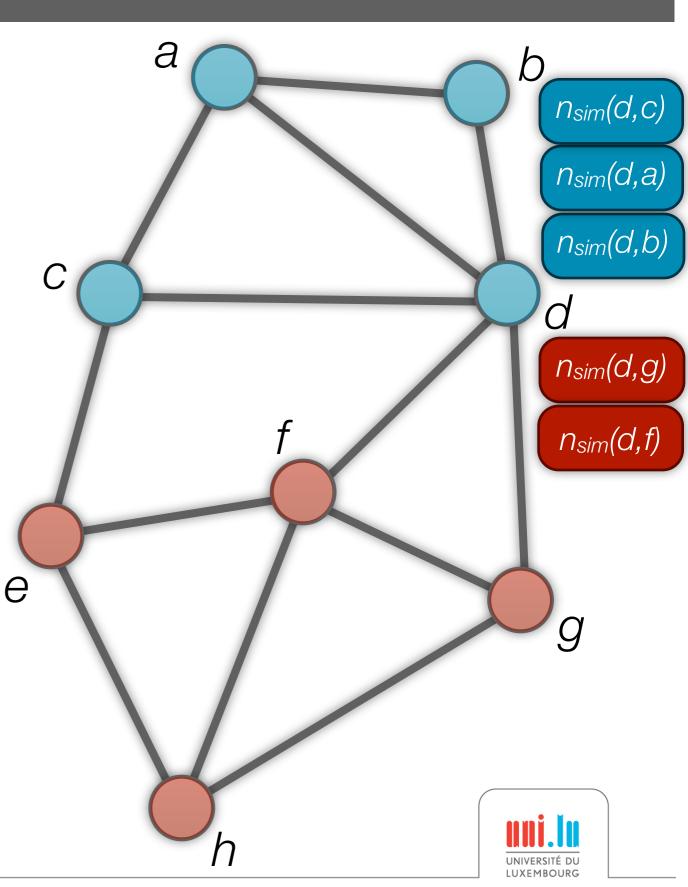


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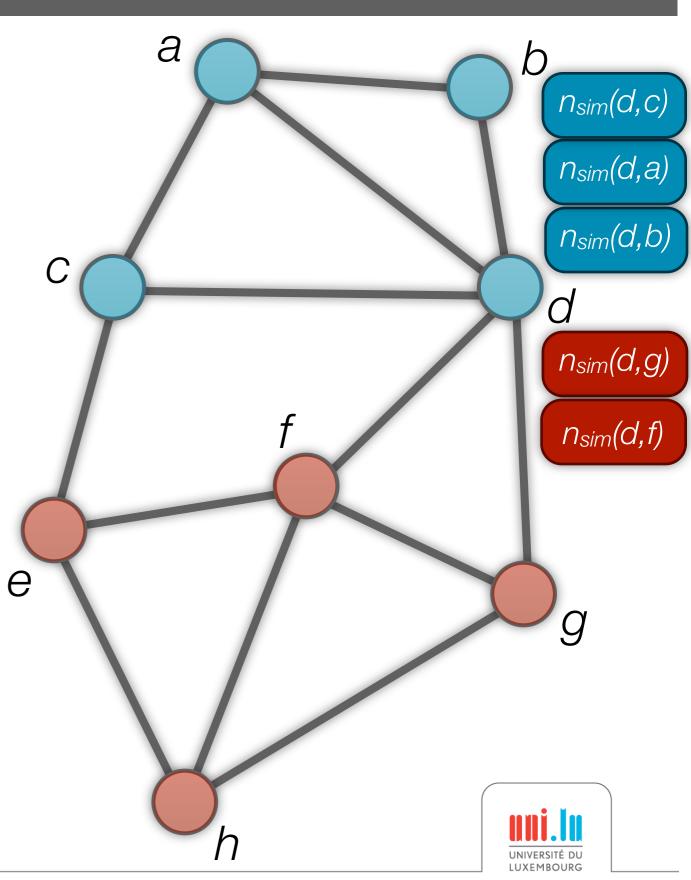
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Objectives:

- Strengthen the community boundaries
- Be more deterministic

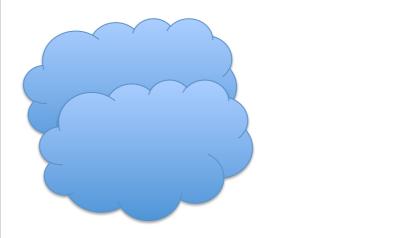




Focus on dynamic networks:

 Break mode: detects split of a community in two disconnected subset

→ prevent the *wandering community* effect



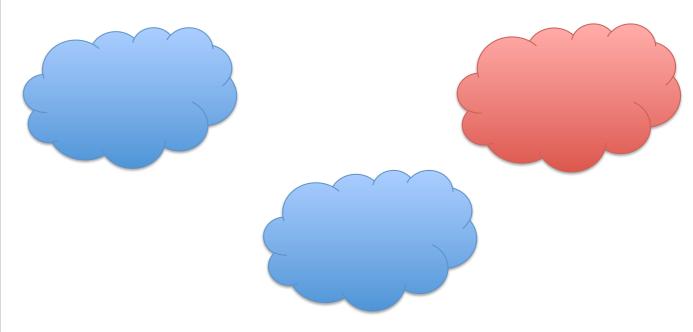




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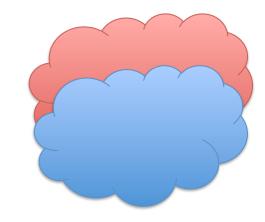


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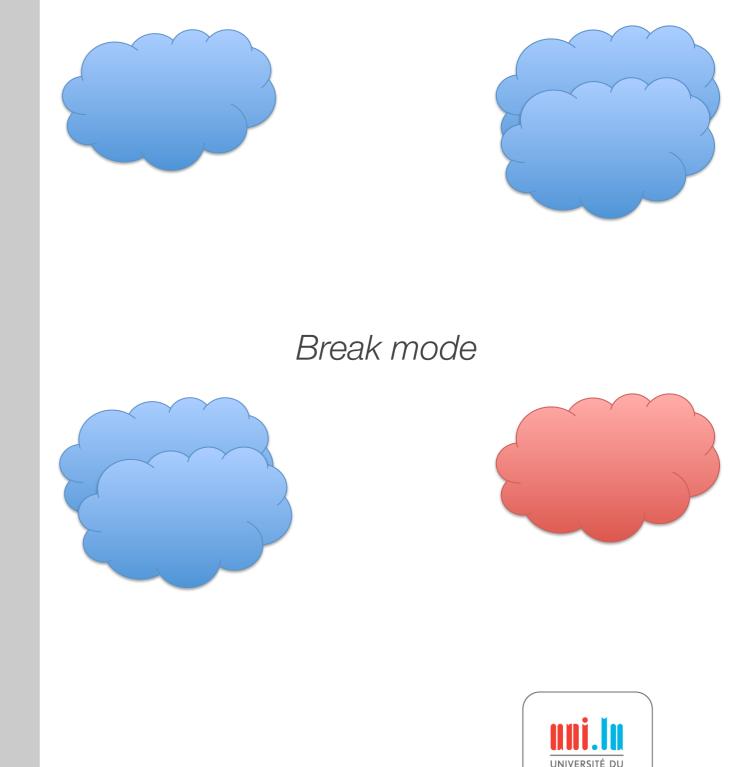


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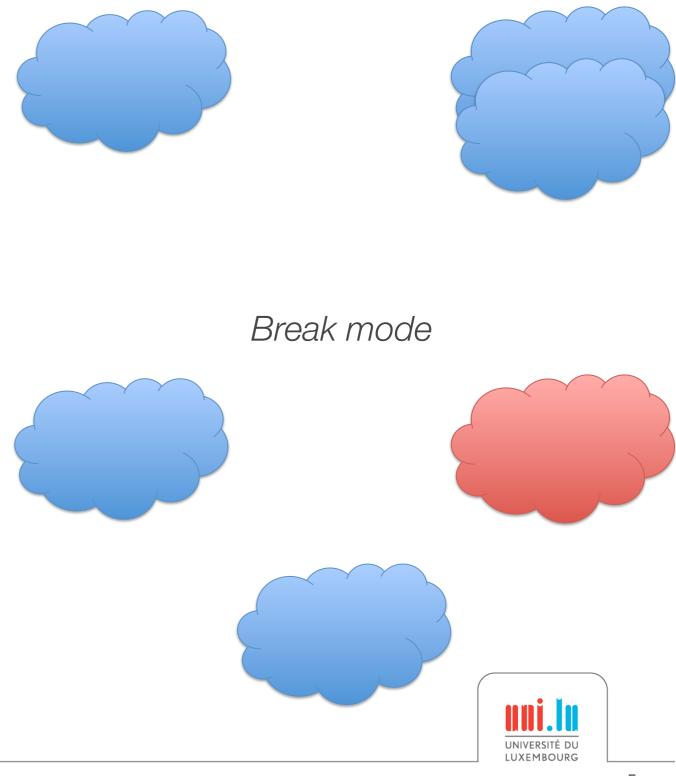


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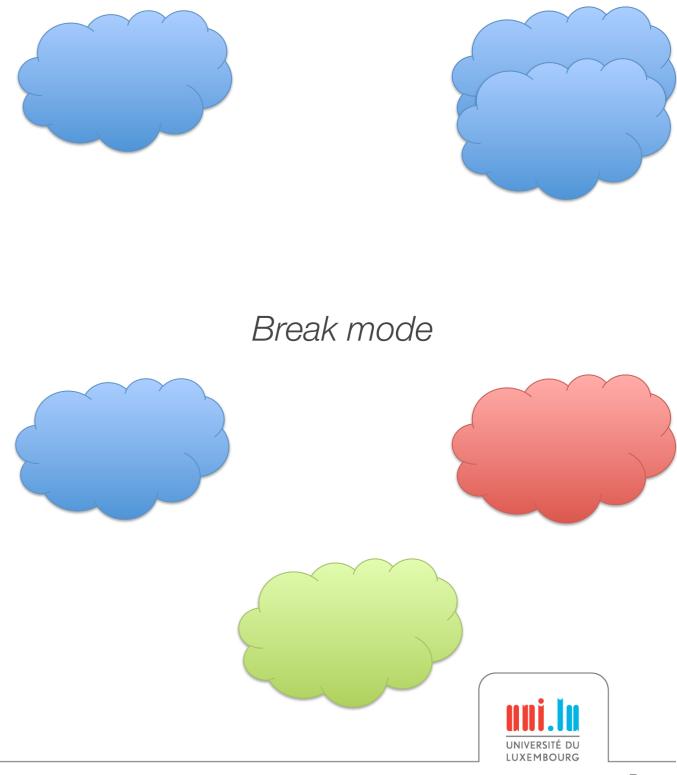
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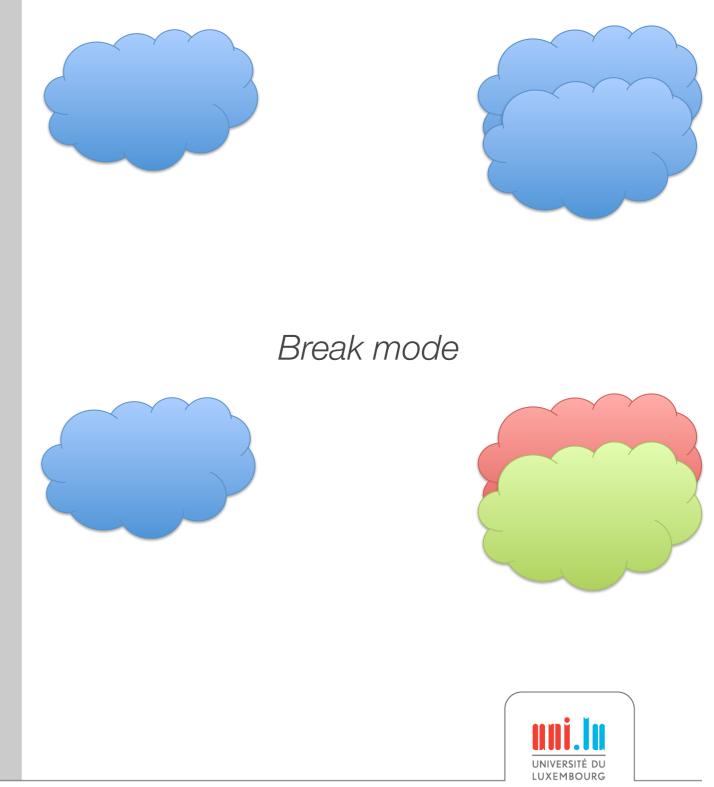
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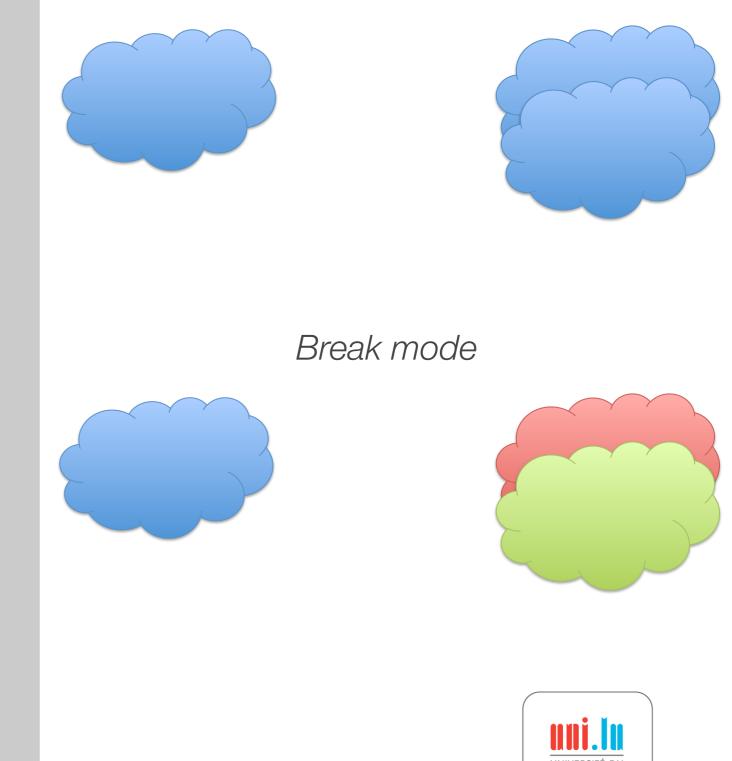


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 Community size limitation: nodes farther away than *d* hops create a new community
 → can be helpful for delayconstrained applications (e.g. VANET safety)

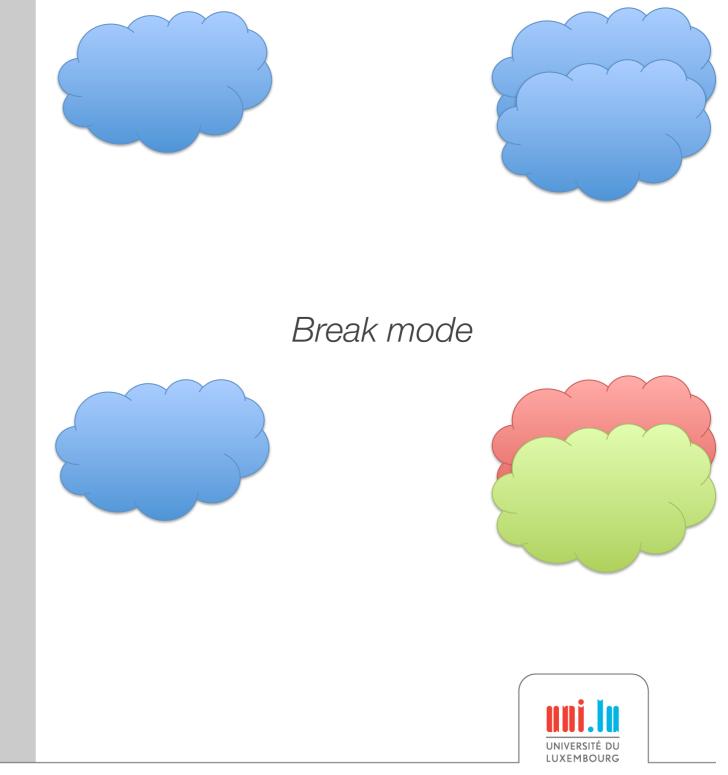


Focus on dynamic networks:

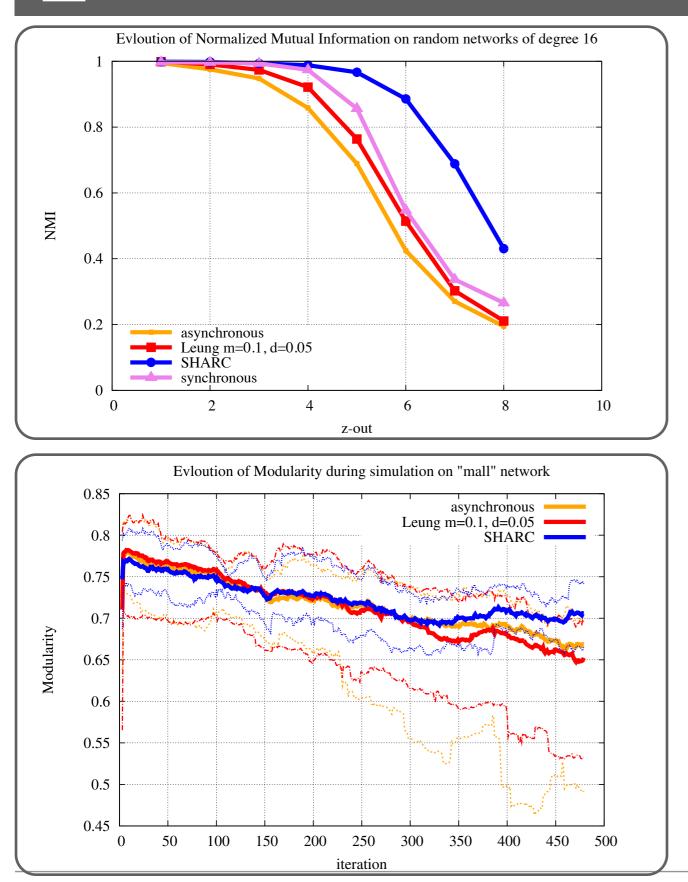
 Break mode: detects split of a community in two disconnected subset

→ prevent the *wandering community* effect

- Community size limitation: nodes farther away than *d* hops create a new community
   → can be helpful for delayconstrained applications (e.g. VANET safety)
- Shift from pure formal considerations to applicative constraints



## **SHARC** performances



- Sharper assignment :
  - Higher modularity Q and NMI
  - Limits jumbo community effect by bounding community size
  - Important for highly dynamic networks
- More robust assignment :
  - Less standard deviation in results (topology, protocol seed)
- Results submitted as conference paper in Dec. 2009 (WoWMoM)
  - Acceptance pending





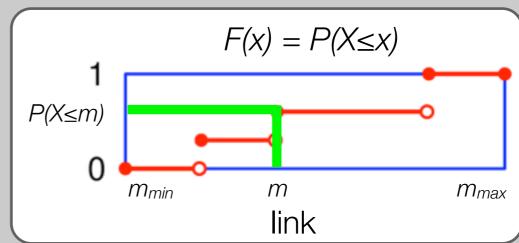
Consider link-stability as edge weight in community assignment process

- Normalized stability estimator (used to modulate the n<sub>sim</sub> metric)
- Auto-adaptive to current network conditions
- Independent from underlying metric: age, average age, SNR (if X-layer), ...



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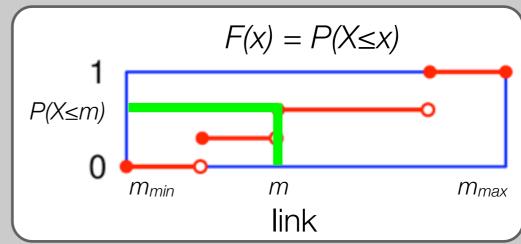
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  - Gives the ratio of edges whose stability is equal or below the current edge value

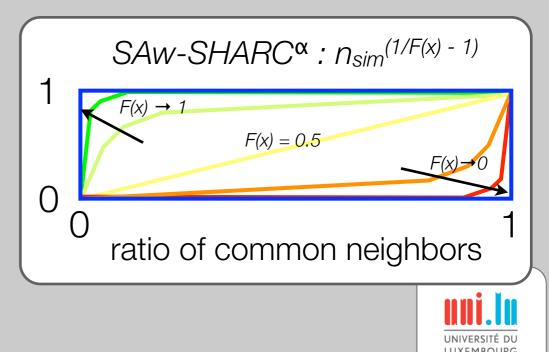




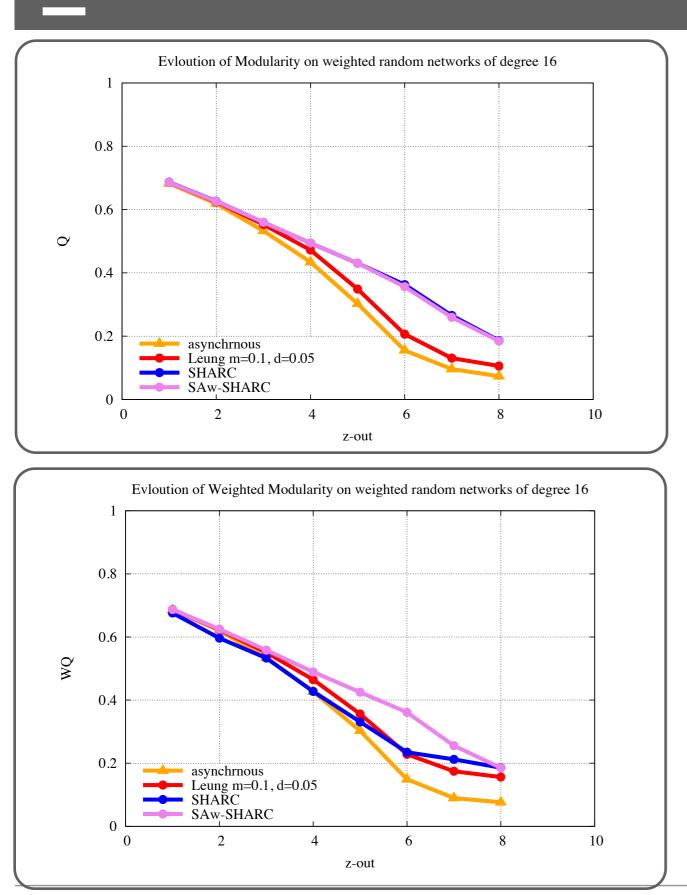
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- CDF is used as modulation exponent of the neighborhood similarity measure
  - Neighbors with stable links contribute more to their community label score
  - Similar notions as in the α model of Watts and Strogatz for small-world





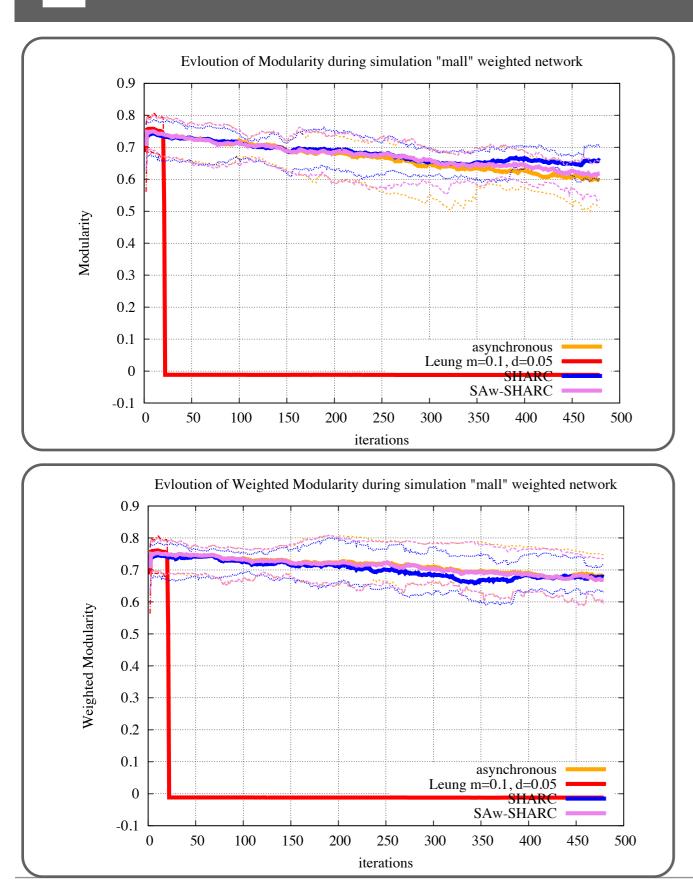
## SAw-SHARC performances (static networks)



- Random networks with varying z-out and uniformly distributed edge weight
- SHARC and SAw-SHARC modularity Q are equivalent
- SAw-SHARC yields better results in terms of weighted modularity WQ
  - especially when compared with other weight-aware algorithms



### SAw-SHARC performances (dynamic networks)



- "Mall" scenario with edge lifetime as quality metric
- SAw-SHARC performs good in terms of modularity Q and weighted modularity WQ
- Different trade-off between unweighted and weighted metrics
- Strange behavior for one of the algorithms on dynamic networks
- Need to investigate under more aggressive dynamic networks



- Use edge weight to represent shared content/interest between nodes
  - Application to MoSoNets (Mobile Social Networks)
  - Need a function to express the interest as a scalar (Locality Sensitive Hashing ?)
  - Maybe a mixed approach stability+interest in a multi-objective problem
- Introduce long-term interaction metrics to the assignment process
  - Favor people with regular interaction pattern to be grouped in the same community
- Use of community structures to help algorithms with scalability
  - Experiments on the tree/community structures matching
  - Scalability in k(connected)-m(dominant)-CDS creation
- Use of community structures for reputation and trust
  - Can we derive (partially ?) node reputation from its community reputation ?
  - Rough idea, further investigation needed

## **Contact information**

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### Thanks for your attention







Considering a network 
$$\mathcal{N}=(E,V)$$



Considering a network  $\mathcal{N} = (E, V)$ We define the adjacency matrix A  $a_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are adjacent} \\ 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$ 



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• Then 
$$\forall i \in V, \forall k \in \mathcal{C}$$
  

$$s_{ik} = \sum_{j \in V} a_{ij} c_{jk} \left( 1 - \frac{\sum_{p \in V} a_{ip} (1 - a_{jp}) + \sum_{q \in V} a_{jq} (1 - a_{iq})}{\sum_{n \in V} a_{in} + \sum_{m \in V} a_{jm}} \right)$$



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• And  

$$c_{ik} = \begin{cases} 1 & \text{if } s_{ik} = max_{c \in \mathcal{C}}(s_{ic}) \\ 0 & \text{otherwise} \end{cases}$$

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- Mathematically, the community assignment problem sums up to computing the assignment matrix C at each iteration step
- Problem is neither quadratic nor linear
- Proof of convergence is hard
- Need to investigate for mathematical programming model

